Encuentre la derivada que se indica:

$$\boldsymbol{a)\ f(x) = e^{x^2}\tan^{-1}\left(e^{x^2}\right)}$$

$$f'(x) = e^{x^2}\frac{d}{dx}\tan^{-1}\left(e^{x^2}\right) + \tan^{-1}\left(e^{x^2}\right)\frac{d}{dx}e^{x^2}$$

$$= e^{x^2}\cdot\frac{1}{1+\left(e^{x^2}\right)^2}\frac{d}{dx}e^{x^2} + \tan^{-1}\left(e^{x^2}\right)\cdot e^{x^2}\cdot\frac{d}{dx}x^2$$

$$= \frac{e^{x^2}}{1+e^{2x^2}}\cdot e^{x^2}\frac{d}{dx}x^2 \ + e^{x^2}\tan^{-1}\left(e^{x^2}\right)\cdot 2x$$

$$= \frac{e^{x^2}}{1+e^{2x^2}}\cdot e^{x^2}\cdot 2x \ + 2xe^{x^2}\tan^{-1}\left(e^{x^2}\right)$$

$$= \boxed{2xe^{x^2}\left(\frac{e^{x^2}}{1+e^{2x^2}} + \tan^{-1}\left(e^{x^2}\right)\right)}$$

$$\boldsymbol{b)\ f(x) = x^2 e^{x^2} \sin\left(e^{x^2}\right)}$$

$$f'(x) = x^2 e^{x^2} \frac{d}{dx}\sin\left(e^{x^2}\right) + x^2 \sin\left(e^{x^2}\right)\frac{d}{dx}e^{x^2} + e^{x^2}\sin\left(e^{x^2}\right)\frac{d}{dx}x^2$$

$$=$$

$$x^2 e^{x^2} \cdot \cos\left(e^{x^2}\right)\frac{d}{dx}e^{x^2} + x^2 \sin\left(e^{x^2}\right)e^{x^2}\frac{d}{dx}x^2 + e^{x^2}\sin\left(e^{x^2}\right)\cdot 2x =$$

$$x^2 e^{x^2}\cos\left(e^{x^2}\right)e^{x^2}\frac{d}{dx}x^2 + x^2\sin\left(e^{x^2}\right)e^{x^2}\cdot 2x + e^{x^2}\sin\left(e^{x^2}\right)\cdot 2x$$

$$x^2 e^{x^2}\cos\left(e^{x^2}\right)e^{x^2}\cdot 2x + x^2\sin\left(e^{x^2}\right)e^{x^2}\cdot 2x + e^{x^2}\sin\left(e^{x^2}\right)\cdot 2x$$

$$= \boxed{\left(2xe^{x^2}\right)\left(x^2 e^{x^2}\cos\left(e^{x^2}\right) + x^2\sin\left(e^{x^2}\right) + \sin\left(e^{x^2}\right)\right)}$$

$$\boldsymbol{c)\ f(x) = \frac{\cot\left(e^{x^3}\right)}{\cot^{-1}\left(e^{x^3}\right)}}$$

$$f'(x) = \frac{\cot^{-1}\left(e^{x^3}\right)\frac{d}{dx}\cot\left(e^{x^3}\right) - \cot\left(e^{x^3}\right)\frac{d}{dx}\cot^{-1}\left(e^{x^3}\right)}{\left(\cot^{-1}\left(e^{x^3}\right)\right)^2}$$

$$= \frac{\cot^{-1}\left(e^{x^3}\right)\cdot -\csc^2\left(e^{x^3}\right)\frac{d}{dx}e^{x^3} - \cot\left(e^{x^3}\right)\cdot -\frac{1}{1+\left(e^{x^3}\right)^2}\frac{d}{dx}e^{x^3}}{\left(\cot^{-1}\left(e^{x^3}\right)\right)^2}$$

$$\frac{-\cot^{-1}\left(e^{x^3}\right)\csc^2\left(e^{x^3}\right)e^{x^3}\frac{d}{dx}x^3 + \frac{\cot\left(e^{x^3}\right)}{1+e^{2x^3}}\cdot e^{x^3}\frac{d}{dx}x^3}{\left(\cot^{-1}\left(e^{x^3}\right)\right)^2}$$

$$= \frac{-\cot^{-1}\left(e^{x^3}\right)\csc^2\left(e^{x^3}\right)e^{x^3}\cdot 3x^2 + \frac{e^{x^3}\cot\left(e^{x^3}\right)}{1+e^{2x^3}}\cdot 3x^2}{\left(\cot^{-1}\left(e^{x^3}\right)\right)^2} =$$

$$\frac{\left(3e^{x^3}x^2\right)\left(-\cot^{-1}\left(e^{x^3}\right)\csc^2\left(e^{x^3}\right)+\dfrac{\cot\left(e^{x^3}\right)}{1+e^{2x^3}}\right)}{\left(\cot^{-1}\left(e^{x^3}\right)\right)^2}$$

$$=\frac{\left(3e^{x^3}x^2\right)\left(\dfrac{-\cot^{-1}\left(e^{x^3}\right)\csc^2\left(e^{x^3}\right)\left(1+e^{2x^3}\right)+\cot\left(e^{x^3}\right)}{1+e^{2x^3}}\right)}{\left(\cot^{-1}\left(e^{x^3}\right)\right)^2}$$

$$\boxed{\frac{\left(3e^{x^3}x^2\right)\left(\cot\left(e^{x^3}\right)-\cot^{-1}\left(e^{x^3}\right)\csc^2\left(e^{x^3}\right)\left(1+e^{2x^3}\right)\right)}{\left(1+e^{2x^3}\right)\left(\cot^{-1}\left(e^{x^3}\right)\right)^2}}$$

$$\boldsymbol{d)}\ \boldsymbol{e^x + e^y = \sin(y)}\quad \boldsymbol{\frac{d^2y}{dx^2}}$$

Derivando explicitamente

$$e^x + e^y \frac{dy}{dx} = \cos(y)\frac{dy}{dx}$$

$$e^x = \cos(y)\frac{dy}{dx} - e^y \frac{dy}{dx}$$

$$\frac{dy}{dx}(\cos(y) - e^y) = e^x$$

$$\frac{dy}{dx} = \frac{e^x}{\cos(y) - e^y}$$

Segunda derivada

$$\frac{d^2y}{dx^2} = \frac{(\cos(y) - e^y)\frac{d}{dx}e^x - \frac{d}{dx}(\cos(y) - e^y)}{(\cos(y) - e^y)^2}$$

$$= \frac{(\cos(y) - e^y)e^x - \left(-\sin(y)\frac{dy}{dx} - e^y\frac{dy}{dx}\right)}{(\cos(y) - e^y)^2} =$$

$$\frac{(\cos(y) - e^y)e^x + \sin(y)\frac{dy}{dx} + e^y\frac{dy}{dx}}{(\cos(y) - e^y)^2}$$

Sabiendo que $\frac{dy}{dx} = \frac{e^x}{\cos(y) - e^y}$

$$\boxed{\frac{(\cos(y) - e^y)e^x + \sin(y)\left(\frac{e^x}{\cos(y) - e^y}\right) + e^y\left(\frac{e^x}{\cos(y) - e^y}\right)}{(\cos(y) - e^y)^2}}$$